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B. Sc. Part-II Paper-IV
Physics Hons
Current electricity

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Reciprocity Theorem of Mutual inductance :-

Let There be two closed circuits (or coils) C_1 and C_2 situated close to each other and carrying currents i_1 and i_2 amp respectively. On account of the current i_1 flowing in the coil C_1, a magnetic field is produced in the space surrounding C_1. If B_1 is the magnetic field induction at a small elements δI_1 of the coil C_2, then the force acting on this element is given by

$$\delta F_2 = I_2\, \delta I_2 \times B_1$$
$$= i_2\, \delta I_2 \cdot B_1 \sin\theta\, \hat{r}$$

where $\hat{r}$ is the unit vector perpendicular to the plane containing δI_2 and B_1 and θ is the angle between δI_2 and B_1.

If δr is the displacement of the element δI_2 along the direction of the force, then work done by this force is

$$\delta W_2 = \delta F_2\, \delta r$$
$$= i_2\, \delta I_2\, B_1 \sin\theta \cdot \hat{r} \cdot \delta r$$
$$= i_2 B_1 \sin\theta\, (\delta I_2\, \delta r)$$
$$= i_2 B_1 \sin\theta\, \delta A_2$$

Where small element of area

$$\delta A_2 = \delta I_2 \cdot \delta r$$

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As $B_1 \sin\theta$ represents the component of B_1 normal to the plane of the area Therefore ($B_1 \sin\theta . \delta A_2$) will indicate the change in the magnetic flux linked with the coil C_2.

Hence, $\delta W_2 = i_2 . \delta \Phi_2$

Therefore the total work done in linked the total flux Φ_2 with coil is

$$W_2 = \int i_2 \delta \Phi_2 = i_2 \Phi_2 \quad \text{——} \quad (3)$$

Similarly the total work done in linked flux Φ_1 with the coil C_1 carrying a current i_1 is

$$W_1 = i_1 . \Phi_1 \quad \text{——} \quad (4)$$

But W_1 must be equal to W_2 because if the closed circuits (coil) C_1 and C_2 are seperated to a great distance, then the forces acting on the coils at any instant during the process of seperation must be equal and opposite

i.e. $W_1 = W_2 = W$ (Say)

i.e. $i_1 \Phi_2 = i_2 \Phi_2 \quad \text{——} \quad (5)$

But from equn (1) and (1'), we have

$\Phi_1 = M_{12} i_2$ and $\Phi_2 = M_{21} i_1$

Therefore equⁿ ③ yields

$$i_1 (M_{12} i_2) = i_2 (M_{21} i_1)$$

or $$M_{12} = M_{21}$$

This relation is usually termed as the reciprocity Theorem of mutual inductance. On account of this Theorem mutual inductance between two coils (or circuits) is represented by M in place of M_{12} or M_{21}.

Mutual Inductance of two Given Coils :-

Consider a long air cored Solenoid (Primary Coil) PP of cross-sectional area 'a' having n_1 turns per metre length. A short Secondary coils SS of n_2 turns is wound closely over the central portion of the Primary PP (figure).

If i amp is the current flowing in the Solenoid, then the magnetic field within it in the middle region is

$$B = \mu_0 n_1 i \text{ webers}/m^2.$$

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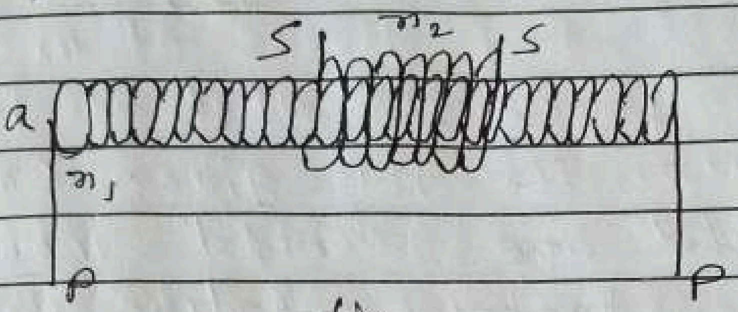
∴ The Magnetic flux linked with each turn of the Central Portion of the Primary is

$$\phi' = Ba = \mu_0 n_1 i \cdot a$$



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Since the Secondary is would closely over the Central Portion of the Primary, the same flux is also linked with each turn of the Secondary i.e. the Magnetic flux linked with each turn of the Secondary is

$$\phi' = \mu_0 n_1 i a \quad \text{—— (1)}$$

∴ Total flux linked with the Secondary having n_2 turn is

$$\phi = n_2 \phi' = \mu_0 n_1 n_2 a i$$

But according to definition if M is the Mutual inductance between the two Coils, the total flux linked with the secondary is

$$\phi = Mi \quad \text{—} \quad (2)$$

Comparing equn (1) and (2), we get

$$M = \mu_0 n_1 n_2 a \quad \text{—} \quad (3)$$

The equn gives the mutual inductance between the two given coils. If the coils are wound round a core of constant relative permeability μ_r, then

$$M = \mu_0 \mu_r n_1 n_2 a \quad \text{—} \quad (4)$$